Two-Country Computable Equilibrium Model of International Drought Risk Sharing: The Case of Pakistan and the Philippines

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Abstract—We formulate a computable equilibrium model to analyze the effects of two drought risk sharing systems on the multi-sector economy of two partner countries. The first system is a proportionally divided system where the crops produced in the two partner countries are proportionally divided between them without extra cost at drought periods. The second system is a forward contract system where the two partner countries agree in advance to export a fixed share of their crops with a fixed price to the partner country in case of drought. In both systems, crops are transferred to the partner country when there is drought only in that country. In the case studies, we focus on Pakistan and Luzon in the Philippines, and quantitatively analyze the effects of the drought risk sharing on the expected agricultural production, the expected GDP, and so on. We conclude by proposing the optimal percentage of allocation in the systems that would benefit both countries and would be fair enough to motivate both of them to sign the drought risk sharing agreement.

Index Terms—food security; international drought risk sharing; two-country open economy model

I. INTRODUCTION

Recently, the uneven distribution of food among developing countries emerged as a serious problem, in addition to the classic “North-South problem” that is characterized by “satiation in the North, and hunger in the South [1].” One factor is that, in the global market, transaction costs among countries are different due to political reasons and to the economic and technological constraints, that determine the activity level of international firms that deliver foreign food to people [2]. Moreover, the increase of the frequency of climate disasters such as droughts and floods makes the problem more severe.

In order to mitigate the regional risk of the lack of food, some international food reserve agreements have already been proposed such as the ASEAN Emergency Rice Reserve (AERR), the ASEAN Plus Three Emergency Rice Reserve (APTEERR) [3], the South Asian Association for Regional Cooperation (SAARC) Food Security Reserve, and the SAARC Food Bank [4]. However, it is pointed out that multinational agreements often face operational difficulties [4]-[6]. One realistic process to follow is to start with bilateral agreements, or use them to complement multilateral agreements.

The relationship between droughts and economic losses has been analyzed by some models such as the Computable General Equilibrium (CGE) model [7]-[10]; the Spacial Computable General Equilibrium (SCGE) model [11]; and the General Circulation Model (GCM) [12]. There are also models which focus on droughts and food reserves using the multi-goods and multi-sector dynamic model [1]; the spatial equilibrium model [13]; and the partial equilibrium model under the imperfect competitive market [14]. An open economy model of risk sharing which consists of two symmetric countries and one sector exists, but it is just for qualitative analysis [15]. The purpose of this research is to construct a two-country multi-sector model to quantitatively analyze the effects of the drought risk sharing agreement in an open economy. As target countries for a case-study, we focus on Pakistan and Luzon in the Philippines, which are vulnerable to drought. We also try to find the optimal percentage of allocation of the agreements that would benefit both countries the most.

The remainder is as follows: Chapter II constructs the base model for the analysis, Chapter III introduces the risk sharing agreement into the model, Chapter IV presents the results of numerical simulation, and Chapter V concludes this research.

II. MODEL

A. Economic Space

The economic space consists of two countries \( i \in I = \{\text{Pakistan, Luzon in the Philippines}\} \) and the Rest Of the World (regarded as one country, ROW), and three sectors \( j \in J = \{a, m, s\} \): agriculture \((j = a)\), manufacture \((j = m)\), and service \((j = s)\). Each sector is considered to be in a perfectly competitive market under constant-returns-to-scale technology and produces perfectly substitutable goods within the same sector. Agricultural and manufactured goods can be traded among countries, but service cannot. Agricultural goods and
service goods are only consumed, while manufactured goods are both consumed and stockpiled as capital stocks. Trade costs are assumed to be included various types of transaction costs such as transportation costs, adjustment costs, and so on. A representative household supplies its labor, capital, and land to firms, and obtains wage, capital rent, and land rent in return from firms. Labor and capital are mobile among sectors within each country, but immobile among countries. Land is classified into two types; irrigated land (\(g = 1\)) and rainfed land (\(g = R\)); and used only in the agricultural sector. The two countries can sign the risk sharing agreement between them if both of them benefit from it.

**B. Drought Occurrence Pattern**

We assume that there are four precipitation patterns between the two partner countries: (a) No drought in the two countries (\(z = 1\)), (b) Drought only in the partner country (\(z = 2\)), (c) drought only in the own country (\(z = 3\)), and (d) drought in both countries (\(z = 4\)). The risk sharing system works effectively when drought occurs only in one country.

**C. Risk Sharing System**

We consider two types of drought risk sharing systems: a “proportionally divided system” and a “forward contract system,” which are implemented in the states, \(z = 2, 3, 4\). In the case of the proportionally divided system, we assume that the crops of both countries are proportionally divided between them without any cost except for the trade costs. In the case of the forward contract system, we assume that one country supplies a certain amount of crops to the other country with a cheaper price than the effective price that it pays to buy from the ROW, which includes the trade costs.

**D. Event Sequence**

The event sequence consists of an ex-ante problem and an ex-post problem as follows:

a) At first, the two countries decide whether they want to sign the risk sharing agreement or not.

b) In the ex-ante problem, by considering the occurrence probability of drought, a representative firm in the agricultural sector determines the optimal input level to cultivate agricultural land in order to maximize their expected profits.

c) The amount of precipitation is determined. In the case of little precipitation, drought occurs.

d) In the ex-post problem, through production factor market and goods-services market, the production and consumption of each good are decided so that all the firms maximize their profits or minimize their production costs, and a representative household maximizes its utility. If the risk sharing agreement is signed, agricultural goods are traded between the own and the partner country in accordance with the contract signed in advance.

e) At the end of the period, the representative household accumulates its capital stocks in preparation for consumption in the future.

**E. Household’s Problem**

1) Utility: A representative household in country \(i\) obtains a utility \(u^z_i\) from the consumption of agricultural goods \(x^z_{ai}\), manufactured goods \(x^z_{mi}\), service goods \(x^z_{si}\), and the capital stocks of next period \(k^{z+}_i\) where \(z\) corresponds to the drought occurrence patterns mentioned in the section II-B. The utility function \(u^z_i\) is represented by

\[
u^z_i := \psi_{x^z_i} \left[ \prod_{j \in \{a, m, s\}} (x^z_{ji})^{\gamma_j} \right]^{1-\theta} - 1 + \psi_{k^z_i} (k^{z+}_i)^{1-\theta} - 1 \frac{1}{1 - \theta}
\]

for all \(i\), where \(\sum_{k \in \{x, k\}} \psi_{bs} = 1\), \(\sum_{j \in \{a, m, s\}} \gamma_j = 1\), and \(\psi_{bs}, \gamma_j \in (0, 1)\). Here, \(\psi_{bs}\) is the consumption-stock share parameter and \(\gamma_j\) is the consumption share parameter of each type of goods. \(\theta\) represents the degree of relative risk aversion.

2) Demand: The demand of agricultural goods and manufactured goods is classified into domestic goods \(q^z_{ai}\), imported goods from the partner county \(d^z_{ai}\), and from the ROW \(\eta^z_{ai}\):

\[
x^z_{ji} = q^z_{ji} + d^z_{ji} + \eta^z_{ji}
\]

for all \(i\) and \(j = a, m\). When we assume that \(\Xi^z_{ai} = [0, 1]\) and \(\Theta^z_{ai} \in [0, 1]\) are the import shares accounted for the national demand \(x^z_{ji}\) from the partner country and the ROW, respectively, from (2), we have

\[
q^z_{ji} = (1 - \Xi^z_{ji})(1 - \Theta^z_{ji})x^z_{ji},
\]

\[
d^z_{ji} = \Xi^z_{ji}(1 - \Theta^z_{ji})x^z_{ji},
\]

\[
\eta^z_{ji} = \Theta^z_{ji}x^z_{ji}
\]

for all \(i\) and \(j = a, m\). As service goods are non-tradable, \(\Xi^z_{si} = \Theta^z_{si} = 0\), \(d^z_{si} = \eta^z_{si} = 0\), and \(x^z_{si} = q^z_{si}\) for all \(i\).

From (3)-(5), we define the effective prices \(p^z_{ai}\) of agricultural goods and manufactured goods which include costs of transaction and transportation as follows:

\[
p^z_{ji} = \tilde{p}_j (q^z_{ji} + \tau^d_{ji}d^z_{ji} + \tau^\eta_{ji}\eta^z_{ji}) = p^z_{ji} x^z_{ji}
\]

for all \(i\) and \(j = a, m\), where \(\tilde{p}_j\) represents the world price of each goods. \(\tau^d_{ji} \in (1, \tau^\eta_{ji})\) and \(\tau^\eta_{ji} \in (\tau^d_{ji}, \infty)\) are trade mark-up ratios that reflect transportation costs with the partner country and the ROW, respectively. So we have

\[
p^z_{ji} = \tilde{p}_j [(1 - \Xi^z_{ji})(1 - \Theta^z_{ji}) + \tau^d_{ji}\Xi^z_{ji}(1 - \Theta^z_{ji}) + \tau^\eta_{ji}\Theta^z_{ji}]
\]

for all \(i\) and \(j = a, m\), where \(p^z_{ji} \in [\tilde{p}_j, \tilde{p}_j \tau^\eta_{ji}]\). Since the service goods are non-tradable, \(p^z_{si} = p^z_{si}\) for all \(i\), where \(p^z_{si}\) means the price of service goods within the country.

3) Utility maximization: We consider that the households follow an optimal behavior after the precipitation pattern \(z\) is determined. From (1), the ex-post utility maximization problem under the budget constraint is represented by

\[
V^z_i = \max_{x^z_i, k^{z+}_i} u^z_i
\]

s.t.

\[
\sum_{j \in \{a, m, s\}} p^z_{ji} x^z_{ji} + p^z_{si} k^{z+}_i = y^z_i
\]

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where
\[ y^z_i = w^z_i + (\tilde{p}_m + r^z_i)(k_i - k^z_i) + (\tilde{p}_m + r_T^z_i)k_T^z_i + \pi^z_{ai}, \quad (10) \]
\[ \pi^z_{ai} = \frac{\Pi^z_{ai}}{N_i} \quad (11) \]
for all \( i \). Here, \( V^z_i \) is an indirect utility, \( x_i = (x^z_{s1i}, x^z_{s2i}, x^z_{s3i}) \) represents the consumption vector, and \( y^z_i \) means the total income of the representative household. \( w^z_i, r^z_i, \) and \( \pi^z_{ai} \) represent wage rate, nominal capital rent, and land rent of agricultural firms, respectively. Then \( k_i \) is the capital stock of initial period and given exogenously, and \( k_T^z_i \) is the land capital stock borrowed from the households with interest rate \( r_T^z_i \). \( N_i \) represents total population, and \( \Pi^z_{ai} \) is the sum of profits in agricultural sector. By solving the above utility maximization problem, we can obtain the optimal level of consumption \( x^*_{ji} \) and the capital stock of next period \( k^*_{ji} \).

**F. Firm’s Problem**

1) **Production technology:** In order to use hydrological data in the model, we introduce the land-water composite \( X^z_{gi} \) [16]. We assume that the land-water composite consists of available agricultural land \( T^z_{gi} \) and available water \( Z^z_{gi} \). Available agricultural land is classified into irrigated land, \( T_{1i} \), and rainfed land, \( T_{2i} \). The land-water composite function \( X^z_{gi} \) and available agricultural land \( T^z_{gi} \) function which are assumed by the constant elasticity of substitution (CES) technology and Cobb-Douglas technology are represented by

\[ X^z_{gi} := \left[ \beta_{T^z_{gi}}(T^z_{gi})^{\alpha_{X^z_{gi}}} + \beta_{Z^z_{gi}}(Z^z_{gi})^{\alpha_{X^z_{gi}}} \right]^{\frac{1}{\alpha_{X^z_{gi}}}}, \quad (12) \]
\[ T^z_{gi} := \kappa_{gi} \left( K^T_{gi} \right)^{\alpha_{T^z_{gi}}} \left( T^0_{gi} \right)^{1-\alpha_{T^z_{gi}}} \quad (13) \]
for all \( i, g \), respectively. Here, \( 1/(1 - \alpha_{X^z_{gi}}) \) is the elasticity of substitution between agricultural land and available water. Then \( \beta_{T^z_{gi}} \) and \( \beta_{Z^z_{gi}} \) are the parameters to show the contribution of agricultural land and available water to the production of the land-water composite, respectively. \( \kappa_{gi} \) is the scale parameter of the land cultivation technology, \( K^T_{gi} \) is the total land capital stock to cultivate agricultural land, \( T^0_{gi} \) is the available land area, and \( \alpha_{X^z_{gi}} \in (0,1) \) is the share parameter between \( K^T_{gi} \) and \( T^0_{gi} \).

A representative firm in the agricultural sector requires labor \( L^z_{ai} \), capital \( K^z_{ji} \) and land-water composite of each type of agricultural land \( X^z_{gi} \) for its production. The production \( Y^z_{ai} \) and the value-added function \( F^z_{ai} \) of agricultural goods are assumed to follow the Leontief technology and the Cobb-Douglas technology, respectively. Namely,

\[ Y^z_{ai} := \min \left[ F^z_{ai}(L^z_{ai}, X^z_{gi}) \right], \quad (14) \]
\[ F^z_{ai} := B_{ai}(L^z_{ai})^{\beta_{ai}}(K^z_{ji})^{\beta_{ki}} \left( X^z_{gi} \right)^{\beta_{xki}} \quad (15) \]
for all \( i \), where \( \sum_{bi \in \{L_{ai}, K_{ai}, X_{ai}, R_{ai}\}} \beta_{bi} = 1 \) and \( \beta_{bi} \in (0,1) \). Here, \( \phi^j_{gi} \) is the input coefficient of intermediate goods \( j' \) which is demanded by \( j \) sector in country \( i \), and \( \gamma^z_{aj} \) represents the intermediate goods of each sector. \( B_{ai} \) is the total factor productivity. Then \( \beta_{bi} \) is the share parameter of the contribution of labor, capital, and land-water composite of each land type to the production of agricultural goods.

We assume that all the firms in the manufacturing sector and the service sector require only labor \( L^z_{ai} \) and capital \( K^z_{ji} \) for their production. The Leontief technology and the CES technology are assumed for the production \( Y^z_{ji} \) and the value-added function \( F^z_{ji} \) of each goods, respectively:

\[ Y^z_{ji} := \min \left[ F^z_{ji}(L^z_{ji}, Z^z_{ji}) \right], \quad (16) \]
\[ F^z_{ji} := B_{ji}(L^z_{ji})^{\beta_{Lji}}(Z^z_{ji})^{\beta_{Zji}} \left( K^z_{ji} \right)^{\beta_{Kji}} \quad (17) \]
for all \( i \) and \( j = m, s \), where \( \sum_{bi \in \{L, K, X, R\}} \beta_{bi} = 1 \) and \( \beta_{bi} \in (0,1) \). \( B_{ji} \) represents the total factor productivity, and \( 1/(1 - \alpha_j) \) is the elasticity of substitution between labor and capital. \( \beta_{bi} \) is the share parameter of the contribution of labor and capital to the production of each goods.

2) **Profit maximization:** The optimization problem of a representative firm in the agricultural sector consists of two-steps: an ex-ante problem and an ex-post problem. We determine the optimal input level of agricultural land \( T^z_{gi} \) in the ex-ante problem and the optimal input level of labor \( L^z_{ai} \) and capital \( K^z_{ji} \) in the ex-post problem. At first, by solving the ex-post problem, we predict the contingent behavior of a representative firm after precipitation.

A representative firm in the agricultural sector determines the input level of labor \( L^z_{ai} \) and capital \( K^z_{ji} \) to maximize its profit. The ex-post profit function \( \Pi^z_{ai} \) is described by

\[ \Pi^z_{ai} = \max_{L^z_{ai}, K^z_{ji}} \left[ p^w_{ai}F^z_{ai} - w^z_i(L^z_{ai} - r^z_iK^z_{ai}) - r^T_i \sum_{g \in \{1, R\}} K^T_{gi} \right], \quad (18) \]

where

\[ p^w_{ai} = \tilde{p}_a - \sum_{j' \in \{a, m, s\}} \phi^j_{ai} \phi^{j'_{ai}} \quad (19) \]

for all \( i \). Here, \( p^w_{ai} \) means the value-added price of agricultural goods. The fourth term on the right hand side of (18) represents the cultivation cost of agricultural land that is already determined in the ex-ante problem and given as a constant term in the ex-post problem. By solving the ex-post profit maximization problem, we can obtain the optimal input level of labor \( L^z_{ai} \) and capital \( K^z_{ji} \).

In the ex-ante problem, the representative firm in the agricultural sector makes an investment \( K^T_{gi} \) in agricultural land \( T^z_{gi} \) to maximize the expected profit \( \Pi^z_{ai} \) under the consideration of contingent profit \( \Pi^z_{ai} \) and the occurrence probability \( \mu^z \) of each state \( z \). From (18),

\[ \Pi^z_{ai} = \max_{K^T_{gi}, K^z_{ji}} \mathbb{E}[\Pi^z_{ai}] = \max_{K^T_{gi}, K^z_{ji}} \sum_z \mu^z [\Pi^z_{ai}], \quad (20) \]
s.t. (13) for all \( i \), where \( \sum_z \mu^z = 1 \) and \( \mu^z \in (0,1) \). Here, \( \mathbb{E}[\cdot] \) represents the expectation operator for drought occurrence pattern, \( z \). By solving the ex-ante profit maximization problem, we can obtain the optimal input level of \( K^T_{gi} \).
3) Cost minimization: All the firms in the manufacture sector and the service sector which do not have a fixed factor, and whose production technologies are given by homogeneous functions of labor $L^z_{ji}$ and capital $K^z_{ji}$ determine the input levels in order to minimize the unit production costs $c^z_{ji}$, for all $i$ and $j = m, s$. Solving the cost minimization problem, we can obtain the cost function $c^z_{ji}$, as follows:

$$c^z_{ji} = \min_{L^z_{ji}, K^z_{ji}} w^z_i L^z_{ji} + r^z_i K^z_{ji}$$

(21)

for all $i$ and $j = m, s$. From Shephard’s lemma, since $L^z_{ji} = \partial c^z_{ji}/\partial w^z_i$ and $K^z_{ji} = \partial c^z_{ji}/\partial r^z_i$, we obtain the optimal level of labor $L^*_{ji} = L^z_{ji} \cdot Y^z_{ji}$ and capital $K^*_{ji} = k^z_{ji} \cdot Y^z_{ji}$ to produce manufactured goods and service goods $Y^z_{ji}$.

G. Market equilibrium

Zero-profit conditions of the manufacturing sector and service sector derive:

$$p^z_{ji} = c^z_{ji}$$

(23)

for all $i$ and $j = m, s$.

Labor and capital market clearing conditions hold such that the demand and supply of labor $L^z_{ji}$ and capital $K^z_{ji}$ are balanced by the adjustment of wage rate $w^z_i$ and nominal capital rent $r^z_i$ in the markets:

$$\sum_{j \in \{a, m, s\}} L^z_{ji} = N_i,$$

(24)

$$\sum_{j \in \{a, m, s\}} K^z_{ji} = (k_i - k^T_i)N_i,$$

(25)

$$\sum_{g \in \{I, R\}} K^T_{gi} = k^T_i N_i$$

(26)

for all $i$. Noting that production factor markets are closed in each country.

The demand and supply of agricultural goods and manufactured goods are balanced by the adjustment of import rates from the partner country $\Xi^z_{ji}$ and the ROW $\Theta^z_{ji}$, and the service price $p^z_{ji}$ in their markets.

Agricultural goods: $(1 - \Xi^z_{ai}) (1 - \Theta^z_{ai}) A^z_i$

$$+ \tau^z_i [1 - \Xi^z_{ai} (1 - \Theta^z_{ai}) A^z_i + \Theta^z_{ai} Y^z_{ai}] = Y^z_{ai},$$

(27)

where $A^z_i := x^z_{ai} N_i + \sum_{j \in \{a, m, s\}} \gamma^z_{a,ji}$, $i$.

Manufactured goods: $(1 - \Xi^z_{mi})(1 - \Theta^z_{mi}) M^z_i$

$$+ \tau^z_i \Xi^z_{mi} (1 - \Theta^z_{mi}) M^z_i + \xi^z_{mi} = Y^z_{mi} + k_i N_i,$$

(28)

where $M^z_i := (x^z_{mi} + k^z_i) N_i + \sum_{j \in \{a, m, s\}} Y^z_{m,ji}$.

Service goods: $x^z_{si} N_i + \sum_{j \in \{a, m, s\}} Y^z_{s,ji} = Y^z_{si}$

(29)

for all $i$, where export goods to the ROW satisfy $\xi^z_{ji} \in [0, Y^z_{ji})$ for $j = a, m$. Note that the notation $i$ represents the partner country of the country $i$.

H. International Trade Balance

Under equilibrium conditions, the trade balance between countries is equal to zero. Assuming that households pay the transportation costs of import goods instead of firms, the trade balance with the partner country, $TB^d_i$, and with the ROW, $TB^q_i$, are represented by

$$TB^d_i := \tilde{p}_m \tau^d_i \Xi^z_{ai} (1 - \Theta^z_{ai}) A^z_i - \Xi^z_{ai} (1 - \Theta^z_{ai}) A^z_i$$

$$+ \tilde{p}_m \tau^d_i (1 - \Theta^z_{mi}) M^z_i - \Xi^z_{mi} (1 - \Theta^z_{mi}) M^z_i = 0,$$

(30)

$$TB^q_i := \tilde{p}_a (\tilde{z}^a_i - \tau^d_i \Theta^z_{ai} A^z_i) + \tilde{p}_m (\tilde{z}^a_i - \tau^d_i \Theta^z_{mi} M^z_i) = 0,$$

(31)

for all $i$, respectively.

III. RISK SHARING AGREEMENT

A. Proportionally Divided System

When $z = 1$, or 4, agricultural goods are not shared between the two countries using the system and the formulation of the model is the same as the basic model without an agreement. When $z = 2$, or 3, the model formulation is as follows:

1) Profit maximization: In the ex-post problem, the profit maximization in the agricultural sector is described as follows:

$$\Pi^z_{ai} = \max_{L^z_{ai}, K^z_{ai}} p^z_{ai} F^z_{ai} + \tilde{p}_m \tau^d_i \Xi^z_{ai} (1 - \Theta^z_{ai}) A^z_i$$

$$- w^z_i L^z_{ai} - r^z_i K^z_{ai} - r^T_i \sum_{g \in \{I, R\}} K^T_{gi}$$

(32)

for all $i$, where $\sum_{i \in \{a, i\}} \rho^z_i = 1$ and $\rho^z_i \in (0, 1)$. Here, $\tilde{i}$ represents the partner country. In the cases where the own or the partner country have the drought, namely $z = 2$, and 3, each of them will receive the predetermined share $\rho^z_i$ of the agricultural production. When $z = 1$, and 4, $\rho^z_i = \rho^z_i = 0$. The value-added price $p^z_{ai}$ has the same form as (19). Then the second and third terms of (32) work as subsidies and taxes from the partner country, respectively. By solving the ex-post profit maximization problem, we can obtain the optimal input level of labor $L^*_{ai}^z$ and capital $K^*_{ai}^z$. The formulation of the ex-ante problem is given in the same way of (20).

2) Agricultural goods market: The demand and supply of agricultural goods is as follows:

$$\sum_{a,m,s}(1 - \Xi^z_{ai})(1 - \Theta^z_{ai}) A^z_i$$

$$+ \tau^z_i \Xi^z_{ai} (1 - \Theta^z_{ai}) A^z_i + \Theta^z_{ai} Y^z_{ai} = Y^z_{ai},$$

(33)

for all $i$. The total amount of supply depends on the agricultural production of the partner country.

B. Forward Contract System

When $z = 1$, or 4, there is no transfer of forward contract goods between the two countries, and the formulation of the model is the same as the basic model without an agreement. When $z = 2$, or 3, the model formulation is as follows:
1) Profit maximization: In the ex-post problem, the profit maximization in the agricultural sector is described as follows:

\[ \Pi^z_{ai} = \max_{L^z_{ai}, K^z_{ai}} \left[ \sum_{i=1}^n \sum_{j=1}^m \sum_{s=1}^s \left( \rho^z_{ai} y^z_{ai} + \epsilon^z_{ai} \bar{p}_a - \epsilon^z_{ai} \tilde{p} \bar{y}^z_{ai} \right) \right] - w^z_i L^z_{ai} - r^z_i K^z_{ai} - r^T_g \sum_{g \in \{1, R\}} K^T_{gj}, \tag{34} \]

where

\[ \rho^z_{ai} = (1 - \epsilon^z_i) \bar{p}_a + \epsilon^z_i \tilde{p} - \sum_{j' \in \{a, m, s\}} \rho^z_{j'i} \phi_{ai}, \tag{35} \]

for all \( i \), where \( \epsilon^z_i = 0 \) when \( z = 2 \), and \( \epsilon^z_i = 0 \) when \( z = 3 \). Here, \( \epsilon^z_i \) is the crop share of the country \( i \) that is to be given as forward contract goods with the forward contract price \( \tilde{p} \). The second and third terms of (34) represent the benefit and the cost of the forward contract.

2) Agricultural goods market: The demand and supply of agricultural goods is as follows:

\[ (1 - \Xi^z_{ai})(1 - \Theta^z_{ai})A^i_0 + \tau^d \Xi^z_{ai}(1 - \Theta^z_{ai})A^i_1 + \epsilon^z_i Y^z_{ai} = (1 - \epsilon^z_i)Y^z_{ai} + \frac{\epsilon^z_i}{\tau^d} \bar{y}^z_{ai} \tag{36} \]

for all \( i \), where \( \epsilon^z_i = 0 \) when \( z = 2 \), and \( \epsilon^z_i = 0 \) when \( z = 3 \). The total amount of supply in the own country decreases when the forward contract goods are donated to the partner country, while it increases when it receives from the partner country.

IV. Numerical Simulation

A. Basic Setup

We implement a case study using real data from Pakistan and Luzon in the Philippines as follows. Leaf Area Index (LAI) [17]-[18] is used as a proxy variable of the land-water composite goods, and the land-water composite function of Pakistan and Luzon in the Philippines is calibrated by the previous study [19] and by the hydrological data of the main island of Luzon in the Philippines given by the AgriCLVDAS model, respectively. The other functions are calibrated using the Social Accounting Matrix (SAM) data of the year 2007 for Pakistan [20] and that of the year 2012 for the Philippines [21]. The SAM of Luzon in the Philippines is processed from the national SAM using the sector GDP data by regions [22]. The population and GDP data are based on the World Development Indicators (WDI) [23] and the capital stock is the data of FRED [24]. We also set the following: parameters: degree of relative risk aversion \( \theta = 1.5 \), the ex-ante interest rate \( r^T_j = 0.09 \), the trade costs with the partner country \( \tau^d_j = 1.05 \), and the trade costs with the ROW \( \tau^h_j = 1.25 \). The drought occurrence probabilities \( \mu^z \) for the four considered patterns \( z \) are assumed as follows: \( \mu^z \in \{\mu^1, \mu^2, \mu^3, \mu^4\} = \{75\%, 10\%, 10\%, 5\%\} \).

B. Results and Discussion

Fig. 1 and Fig. 2 are the simulation results of the expected agricultural production and the expected GDP in the case of the proportionally divided system and the forward contract system, respectively, and indicate the rates of changes of the expected agricultural production and the expected GDP of both countries corresponding to changes of the shares that the both countries are allocated crops at drought periods. Note that values of the vertical axis of each graph are given by the ratios of the above variables under the agreement to the corresponding values without the agreement.

1) Proportionally divided system: The expected agricultural production increases by 0.03% in Pakistan and by 0.12% in Luzon in the Philippines if the agreement proportions are set to (80%, 20%). The reason why this proportion is better than the others is supported by a fact that the original ratios of agricultural production of each country to the sum of those for the two countries are 82% for Pakistan, and 18% for Luzon in the Philippines, respectively [20]-[22]. Then, when the agreed proportion is (80%, 20%), the consumption of agricultural goods in Pakistan when only Pakistan is faced with drought increases by 1.3%, and the consumption of agricultural goods in Luzon in the Philippines when only the Philippines is faced with drought increases by 2.6%.

Moreover, in Fig. 1(a), we find that there is no hold-up problem in this case study, namely, the incentive of production for each country is not decreasing with the decrease of the proportion. Instead, their incentive of production is decreasing with the increase of the proportion. It implies that “a gift effect” is dominant in agricultural production. When countries receive larger proportions of crops from the partner country, they allocate more production factor to manufacturing and service sectors to increase utility.

2) Forward contract system: We assumed \( \epsilon^z_i = \epsilon^z_j \), whose values are given in the horizontal axes of Fig. 2, and the forward contract price is \( \tilde{p} = 1.1 \). We found that opposite effects emerge in the two countries. Pakistan increases the production of agricultural goods when the percentage of forward contract goods transferred increases because it can sell agricultural goods at a higher price than the international price. On the other hand, Luzon in the Philippines increases the amount of manufactured goods to buy agricultural goods from the partner country. When the agreed percentage of the transfer is 10%, the consumption of agricultural goods in case of drought is increased by 0.3% in Pakistan and by 8.4% in Luzon in the
Philippines. It is also found that the expected GDP of both countries increases with the increase of the percentage of goods transferred in accordance with the forward contract.

V. CONCLUSION

We formulated a two-country multi-sector computable equilibrium model to analyze the effects of the two proposed drought risk sharing systems where the event sequence consists of an ex-ante problem and ex-post problem in an open economy, and conducted a case study of Pakistan and Luzon in the Philippines. In the proportionally divided system, it was found that, while the gift effect emerges in the agricultural production of both countries, there existed proportions that made both countries benefit in terms of the expected GDP. In the forward contract system, it was found that both Pakistan and Luzon in the Philippines had an increase in their expected GDP with the increase of the percentage of goods transferred in accordance with the contract, while the strategies of the two countries were different; Pakistan tried to sell more crops at a higher price, and Luzon in the Philippines produced more manufactured goods to buy crops from Pakistan.

Future tasks include the following; first, more case studies that focus on various types of countries need to be conducted. Second, the subsistence level of consumption in the utility function needs to be considered in order to discuss the starvation problems. Finally, hybrid risk sharing systems could be designed by combining the two systems and investigated the effects.

REFERENCES