Development of a Computational Model based on Particle Swarm Optimization and Network Flow applied to the problem of Hydrothermal Coordination

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Abstract—The hydrothermal coordination can be defined as a problem to determine the optimum usage of the hydroelectric and thermoelectric resources available during a period. In hydrothermal generation systems with a predominance of hydroelectric power plants, like in the Brazilian system, the problem consists in replacing the thermal generation by hydropower generation to minimize the system operational costs. Therefore, this paper presents a model based on Particle Swarm Optimization, and Network Flow applied to the problem of hydrothermal coordination. The goal is to determine an optimal operational strategy for the reservoirs of the hydroelectric power plants considering each hydroelectric power plant separately, its operational constraints and guaranteeing the applicability of the solutions, in order to minimize the operating cost of the system. The proposed approach is compared with four other optimization methods: a Genetic Algorithm (GA), a model based on Genetic Algorithms and the Takagi-Sugeno Fuzzy Inference System (GA+Fuzzy), a model based only on PSO and an optimization algorithm that employs Network Flow and Reduced Gradient (NF+RG). A hydroelectric system composed of three hydroelectric power plants and six different hydrological scenarios were used to test the algorithms. The primary objective was to illustrate the viability and applicability of the proposed algorithm, and, based on the obtained results, show the efficacy and energy gains that are possible.

Keywords - optimization, hydrothermal coordination, particle swarm optimization, network flow.

I. INTRODUCTION

The objective of the hydrothermal coordination is to identify an operational strategy in such a way that the available resources to generate electricity are used in an efficient manner resulting in the decrease of operational costs. [1], [2], [3]. This problem is complex due to a couple of factors like specific unit constraints, the presence stochastic processes like flow rate of the rivers and power demand, the decisions are time coupled, and others. [4].

In hydrothermal power systems with the predominance of hydropower, like the Brazilian system, the operational scheduling aims to favor hydropower generation, whenever possible, over fossil fueled electricity generation and power exchange. This operational scheduling is equivalent to determine an optimal generation schedule so that it is possible to meet the power demand with minimum operational costs during the period for which the generation schedule is being determined. As an outcome, optimum values of the system decision variables such as the volume of water stored in hydroelectric power plants’ reservoirs, turbinated flow, electricity generated by each hydroelectric power plant, the amount of electricity generated by fossil-fueled power plants and energy stored in each power plant and the system as a whole are determined. [5], [6].

This paper shows the results obtained when a hybrid model based on Particle Swarm Optimization (PSO) and Network Flow is used to determine the optimal operational strategy concerning the reservoir levels of each hydroelectric power plant. According to [7], [8], [9], the PSO is a computational model with several advantages such as: ease of implementation; it considers a population of solutions; only a few parameters need to be adjusted; no need to solve derivatives, and it is well-suited to find the global maximum and minimum. The hydrothermal coordination problem was modeled by a network flow with capacitated arcs. This approach is commonly cited in the literature mainly because it explores the special network structure of the problem [4], [5], [10], [11], [12], [13]. This structure was initially proposed by [10], and it was called time-scattered arborescence. It consists of a network where each node has only two outgoing arcs, representing the final water volume in the reservoir and the flow release of the hydroelectric power plants. The objective of the proposed methodology is to assure the feasibility during the whole process for every solution, followed by the individual representation of each hydroelectric power plants, considering important aspects, like the maximum swallowing as a function of the net water head.
The network flow model is based on [5].

To test the efficacy of the proposed model, its performance was compared with the following models: a model that employs network flow and reduced gradient; a PSO algorithm without the network flow model; a Genetic Algorithm and another hybrid model using Genetic Algorithms and a Fuzzy Takagi-Sugeno Inference System. The proposed methodology tries to illustrate how feasible it is to develop and obtain the model, and also the efficacy and energy gains resulting from this model.

The organization of this paper is as follows: Section II explains the mathematical formulation of the problem; Section III shows in detail the proposed approach and describes the problem modeling; Section IV discusses the obtained results and compares them to the other techniques; the final considerations and the conclusion of this paper is present in Section V.

II. MATHEMATICAL FORMULATION

The problem of hydrothermal coordination, with individuated representation of hydroelectric power plants and with deterministic inflows, can be formulated by:

$$\min \sum_{t=1}^{T} = CV P_t \cdot 0.5 \cdot \Phi (E_t)^2 + V (x_T)$$  \hspace{2cm} (1)

s.a.

$$E_t = D_t - H_t,$$

$$H_t = \sum_{i=1}^{N} k_i \cdot h l \left( x_{i,t}^{\text{med}}, u_{i,t} \right) \cdot \min \left[ u_{i,t}, q_{i,t}^{\text{max}} \right],$$  \hspace{2cm} (3)

$$x_{i,t+1} = x_{i,t} + \left( y_{i,t}^{\text{inc}} + \sum_{k \in \Omega_i} u_{k,t} - u_{i,t} - x_{i,t}^{\text{evap}} \right) \left[ \frac{\Delta t_t}{10^6} \right],$$

$$u_{i,t} = q_{i,t} + v_{i,t}.$$  \hspace{2cm} (5)

$$x_{i,t}^{\text{min}} \leq x_{i,t} \leq x_{i,t}^{\text{max}},$$

$$u_{i,t}^{\text{min}} \leq u_{i,t} \leq u_{i,t}^{\text{max}},$$

$$q_{i,t}^{\text{min}} \leq q_{i,t} \leq q_{i,t}^{\text{max}},$$

$$x_{i,0} \text{ given},$$  \hspace{2cm} (9)

where:

- $T$: number of intervals of the planning horizon;
- $N$: number of hydroelectric plants;
- $CV P_t$: coefficient of present value;
- $E_t$: total thermal generation [MW];
- $H_t$: total hydroelectric generation [MW];
- $D_t$: demand (electric energy market) [MW];
- $x_{i,t}$: volume stored in the reservoir $i$ at the end of the interval $t$ [hm$^3$];
- $x_{i,t}^{\text{med}}$: medium volume stored in the reservoir $i$ in the interval $t$ [hm$^3$];
- $x_{i,t}^{\text{evap}}$: volume evaporated in the reservoir $i$ in the interval $t$ [hm$^3$];
- $y_{i,t}^{\text{inc}}$: incremental water inflow to the reservoir of the plant $i$ in the interval $t$ [m$^3$/s];
- $q_{i,t}$: turbinated flow of the plant $i$ in the interval $t$ [m$^3$/s];
- $u_{i,t}$: flow release of the plant $i$ in the interval $t$ [m$^3$/s];
- $v_{i,t}$: spill flow of the plant $i$ during the interval $t$ [m$^3$/s];
- $h_{i,t}$: net water head of the plant $i$ in the interval $t$ [m].

The objective function (1) is composed of two terms, which represent the operational cost during the horizon planning ($\Phi$) and the future costs associated with the final storage condition from hydroelectric plants to reservoirs ($V$). The equality (2) represents the constraints towards the electric power demand in the interval $t$. The equation (3) represents total hydraulic generation of the hydroelectric system. The water balance in the reservoir in the interval $t$ is represented by equation (4). This constraint points out that the final volume is equal to the initial volume, summing the inflow rate (incremental flow rate and release flow of the power plants immediately at upstream) minus the outflow rate (release flow) and evaporated volume. Equation (5) determines the release flow during interval $t$, which is the result of summing turbinated flow rate and spilled flow rate. The inequalities (6), (7) and (8) represent the storage, the released flow and turbinated flow limits of the hydroelectric plants in the interval $t$. The values of the initial volumes of the reservoirs are given (9).

The proposed optimization model (Network Flow and Particle Swarm Optimization) uses a two-dimensional matrix with the final volumes of hydroelectric power plants as model of the position vector of PSO. Based on these final volumes, on the natural flows of the period and on the system demand, the equation 1 is responsible for calculating the operating cost of the system. Therefore, the equation 1 defines the evaluation of each particle of the swarm. Thus, the particles that have the best evaluation are those that are a better energy policy for hydroelectric plants of the test system. The details of modeling the problem are described in section III below.

III. METHODOLOGY

A. Proposed Algorithm

This paper proposes the application of a PSO, in which only a few positions of the velocity and position vectors
change at each iteration. The positions to be adjusted are
determined by the network flow algorithm. This algorithm also
supports the PSO in calculating the updated value of each
position to be modified in a way to assure the feasibility of
the solutions generated by each iteration of the algorithm. The
Figure 1 below summarizes the steps taken by the proposed
model to optimize the hydrothermal coordination, showing
the interaction between the PSO and the network flow (NF + PSO). Right after, each step showed in the picture will be
explained.

1) Particles Initialization: In this step, it is needed to
initialize the position and velocity of the particles. A two-
dimensional matrix $N\times K$ represents the position of a particle
$(p_i)$, where $N$ stands for the number of hydroelectric power
plants and $K$ corresponds to twice the number of interval of
the planning horizon $(2T)$. The matrix contains $2T$ columns,
because the first $T$ columns of the matrix represent the stored
volume of the reservoir $(x_{i,t})$ in each interval $t$, and the second
half of columns refer to the flow release $(u_{i,t})$ of the plants.
Figure 2 shows the model used for the particles of the PSO
implemented for a hydrothermal power system, composed of
four power plants and four months. It is important to highlight
that the velocity vector has the same dimensions as the position
vector. The reason behind the modeling of the velocity and
position vectors is that each node of the network flow has
two outgoing arcs (volume stored in the reservoir and flow
release).

2) Particles Evaluation: The optimization of the problem
aims minimizes the operational cost of the hydrothermal power
system. Therefore, the equation (1) was used as an evaluation
function of the particles of the swarm. The assessment of a
particle in the current iteration is compared to the evaluation
of this same particle in the previous iteration. Case the value
is better than the value in the previous iteration, the position
vector $pBest$ is updated, otherwise, it does not change.

3) Finding gBest: In this step, a search for the particle
with the best fitness in the entire swarm is performed, and the
position vector of this particle is stored in the $gBest$ variable
for the future update of the speed of the swarm particles.

4) Network Flow Part 1: The solution methods, for a
network flow model, use partitions of the entire set of arcs
between basic arcs and non-basic arcs. The basic arcs represent
the dependent variables of the system, whereas the non-basic
arcs represent the independent variables.

To decrease the computational cost in determining the walk
direction of the independent variables, the complete set of non-
basic arcs is partitioned in fixed and super basic arcs. This
way, only a subset of the whole set of non-basic arcs is used.
Thus, the non-basic arcs are divided into two subsets: a set of
non-basic fixed arcs with no walk direction, which means
they are not going to be modified, and a set of arcs that are
going to have their walking directions modified, and for this
reason, they are denominated super basic arcs.

The first step of the network flow is responsible for
determining the super basic arcs, that is, through the arcs
partitioning strategy (APS), the network flow establishes for
each particle, according to its position vector, which positions
of this vector and the speed vector have to be updated. The
details of each strategy are described below:

- **4.1 Arcs Partition Strategy- Hydraulic Production Func-
tion (APS-HPF):** this partitioning strategy recommends,
whenever possible, that the set of basic arcs must be
composed of the volume values. Hence, case a volume arc $(i, t)$ is within its operational limits, this arc is on the
base. Otherwise, the basic arc is going to be a flow release
arc. As the process is interactive, case the value of the arc
volume returns to a value which is within its limits during
the successive iterations; then the arc volume changes its
base to the original one.

- **4.2 Arcs Partition Strategy - Transfer of Energy between
Cascade 1 (APS-TEC1):** this strategy tries to transfer
energy among all the horizon planning periods, that is, it
attempts to move the energy through the entire cascade
(plants of the system) and among all the interval plan,
allowing the transfer of big amounts of energy among
the intervals. The energy transfer is done by creating a
cycle between two intervals, and the PSO modified only
the variables of this cycle.

- **4.2 Arcs Partition Strategy - Transfer of Energy between
Cascade 2 (APS-TEC2):** this APS represents a variation
of the APS-TEC1, being the major difference that this
strategy transfers the energy only among planning horizon
periods defined between the intervals with higher and
lower marginal operational cost. The computational cost
of APS-TEC1 motivated this implementation.

5) Particle Velocity Update: As the PSO works together
with the network flow, the particle velocity update is going
to be applied only to the positions of the velocity vector that
have been determined by the network flow as super basic arcs,
calculated at step 4. Therefore, only the values identified as
super basic arcs are going to be updated. It is important to
highlight that at this point, the $pBest$, which refers to the best
position encountered by the particle, and the $gBest$ which is
the best position experienced by a particle in the whole swarm,
are being used.

6) Network Flow Part 2: In this second part, the following
methods are executed:

- **6.1 Cycles identification:** a cycle occurs when a super
basic arc is included in the structure of the network tree,
when the orientation of the basic arcs of the network may
agree or disagree with the cycle, which is determined by
the direction of the super basic arc. The modification of
the values of a set of arcs helps to calculate the effect
of this amendment on the objective function. Thus, after
the identification of the super basic arcs in step 4 and
the walking direction in step 5, in this step, there is the
determination of the cycle created by the inclusion of the
super basic arc in the tree structure.

- **6.2 Walk Projection of Super Basic Arcs:** this step aims to
prevent the violation of the canalization of the super basic
arcs. There is a need to cancel the walking direction of
6.3 Walking Directions of the Super Basic Arcs: the walking orientation of the basic arcs is calculated based on the cycles constituted by the super basic arcs from which the basic arcs derive. For each cycle, the walking direction of the basic arcs refers to the values given by the combination (addition) of the directions of the super basic arcs, where the basic arcs that have the same orientation of the cycle are positive, or negative, otherwise.

6.4 Maximum Step Calculus: the maximum step is computed to ensure that none of the arcs are going to violate its canalization limits. Following the calculation of the walking direction of the super basic and basic arcs, it is needed to analyze the signal of the walking direction of the decision variable. Depending on the signal, negative or positive, the arc can vary its inferior or superior limits, respectively. This way, for each decision variable, the maximum step is calculated aiming the non-violation of their boundaries. The value of the maximum step for all the decision variables is given by the smallest value among the maximum steps of the variables that correspond to either a basic arc or super basic arc present in at least one cycle.

6.5 Optimum Step Calculus: a unidimensional search such as the golden ratio method determines the value of the optimum step.

7) Particle Position Update: Once the value of the velocity of each position related to the decision variables is calculated, the position of a particle \( p_i \) in the next iteration is established like an additive influence of the old position and the calculated velocity.

IV. RESULTS AND DISCUSSIONS

The warranty of electricity supply requires detailed research on the satisfactory operation of the electrical power systems. The optimization of the hydrothermal coordination is a part of a broad range of studies necessary to meet the requirements of a generation system. This section aims to show the results obtained by applying the optimization model to the hydrothermal coordination problem specified and implemented during the research.

A. Operational Conditions

Six case study were used considering the natural river flow for the time periods from 1936 to 1941; from 1951 to 1956; from 1971 to 1976; from 1980 to 1985, from 2000 to 2005 and considering the long-term average (LTA). The long-term average of a hydroelectric power plant for a given month corresponds to the arithmetic mean of the historical flow rate registered for that power plant. In every simulation, the planning horizon of five years with monthly frequency.
was considered. The month of May was considered as the beginning of the planning horizon, because this month corresponds to the dry season for the southeast river basin, region where hydroelectric power plants are located. The electricity market was kept constant and with the same value of the installed capacity of the hydroelectric system (512 MW). The initial volume in every test case was considered equals to the maximum operational volume.

This work proposes a new approach for the optimization of hydrothermal coordination based on capacitated network flow modeling and optimized by a PSO (NF+PSO). The verification of the quality and consistency of the solutions obtained by the NF+PSO model was obtained by the comparison with another four optimization models: GA, GA + Fuzzy, PSO and NF+RG, the last one was developed by [5]. All of the optimization models used the same operational conditions described in the previous subsection and were tested against six case studies. For each optimization algorithm, ten executions were performed.

B. Results

The case studies using more than one cascade hydroelectric power plant show the change in the behavior of the reservoirs due to its position in the cascade and the operation of other power plants [4], [6], [13].

The resultant volume curve of the optimization given by the proposed algorithm (NF+PSO), considering a scenario with three hydroelectric power plants and 80% of the data of the long-term average is presented in Figure 3. According to the results, it is noticeable the change in the volume of the reservoirs in function of the position of the power plant in the cascade. The Emborcação power plant, positioned at the upstream of the cascade, is responsible for the regulation of the incremental flow rates softening the seasonality and preventing the overflow in the reservoirs. The Itumbiara power plant, located in the middle of the cascade, has fewer oscillations in the volume of its reservoir when compared to the Emborcação power plant, however, it has more severe swings when compared to São Simão power plant. The São Simão power plant is operated with the reservoir in maximum level of volume during almost all the intervals of the planning horizon. This happens because the productivity of the power plants to the downstream of the cascade is what determines the stored energy in the system. Hence, São Simão is operated with its maximum productivity given that it operates during the most of the planning period as a run-of-the-river plant with the maximum volume. This fact is confirmed by the results of other authors [4], [5], [6], [10]. Therefore, the proposed algorithm emphasized the replenishment of the reservoirs from downstream to the upstream part and the emptying of the reservoirs in the opposite direction.

The acute flow release of Emborcação and the softened flow release for Itumbiara happens due to the quota effect. This effect prioritizes the immediate replenishment of the reservoirs, in such a way that they can operate with maximum productivity during the remaining time intervals of the planning horizon. Hence, it is possible to verify that the emptying of the upstream reservoirs is done to prevent overflow and to regulate the flow rates so that, at the end of the rainy season, this reservoir can recover its maximum level of storage [8].

The energy stored in the system (ESS) for the test case scenario using 80% of LTA can be seen in Figure 4. The energy stored in a hydroelectric system composed of a single power plant (i) in a determined planning horizon (t) corresponds to the energy that the power plant can generate to drain out the net volume of its reservoir [5]. Figure 4, shows that the values of the stored energy for each time interval in the planning horizon obtained by the NF+PSO followed the results achieved by NF+RG. Finally, the proposed optimization model was able to reproduce the optimized operational trend of the reservoirs when compared to the trajectories of the stored energy in the system.

![Fig. 3. Trajectories Volume of Plants Emborcação, Itumbiara and São Simão](image1)

![Fig. 4. Comparison of the trajectories of the energy stored in the system for the optimization models NF+PSO, PSO, NF+RG, GA and GA+Fuzzy for LTA 80%](image2)
From Table I, it is possible to affirm that the proposed algorithm was responsible for supplying electricity as economically as the NF+RG hybrid model in all six models. Nonetheless, even though the NF+PSO obtained better results than the other optimization methods, it had not when compared to GA and GA + Fuzzy for the period of 1951 to 1956. These results show that the proposed optimization method can obtain good results when considering three interconnected hydroelectric power plants and different hydrological scenarios. The optimization models GA and GA + Fuzzy had results similar to the NF+PSO approach, but it was more costly, excluding the range from 1951 to 1956.

The results shown in Figure 4 and Table I, indicate that the optimization model that uses only PSO does not improve efficiency when compared to other models, and in special for (NF + PSO). Additionally, the optimized energy operation of the reservoirs given by the PSO did not try to periodically empty or fill the reservoirs during the planning horizon intervals, that means the quota effect did not happen. Moreover, at the end of the planning period, the ESS calculated by the PSO was at its minimum value, whereas other models calculated the maximum ESS. Besides of assuring the feasibility of the solutions, this fact means that the network flow model searches for the optimized energy operation of the system.

The model using network flow guarantees a good computational performance. As a final result, this technique does not impose any numerical or computational limits by representing each power plant individually and a great variety of operational constraints intrinsic to the system.

V. CONCLUSION

This paper presented an optimization model applied to the problem of hydrothermal coordination using network flow and PSO. The research also covered the math formulation and the modeling of the problem. The results obtained using the NF+PSO approach were consistent with the hydrothermal operation found in the literature and showed the applicability of the proposed model considering a system composed of three cascade hydroelectric power plants and different hydrological scenarios. The results also showed that the model based on network flow assured the feasibility of the solutions as well as it respects all of the pertinent constraints of the problem. All the results were coherent and of high quality when compared to the NF+RG model, indicating that the NF+PSO approach tried to maximize the hydroelectric benefits of the hydrothermal generation system considering individualized power plants.

REFERENCES